On the Shapes of the Equipotential Surfaces in the Air near Long Walls or Buildings and on their Effect on the Measurement of Atmospheric Potential Gradients.

By Charles H. Lees, D.Sc., F.R.S.

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§ 1. In considering the most suitable arrangements for recording the variations of the atmospheric potential gradient at the East London College, where no large horizontal surface is available, I have had occasion to calculate the distribution of potential in the neighbourhood of walls and buildings of simple shapes. As the results can be applied in practice it seems advisable to put them on record for the use of other observers.

Observation shows that during fine weather the potential at a point in the atmosphere over a level portion of the earth's surface in these latitudes increases as the point is raised, at the rate of about 150 volts per metre. This rate of increase diminishes slowly as the point ascends, owing to the slight excess of positive over negative ions in the air near the earth's surface, and at an altitude of a kilometre is reduced to about 25 per cent. of its value at the surface.*

The potential differences between points on the earth's surface 1000 metres apart or between a point on the surface of a building and one on the ground near it are found to be small compared to those present in the atmosphere.

For the present purpose we shall neglect the small effect of the ions near the earth's surface on the potential gradient for the first few metres above the surface and shall treat the earth's surface as plane and the earth and buildings as being conductors.† In order further to simplify the problem as much as possible, the walls of the buildings are taken to be long in comparison to their heights, so that the effects of the corners on the distribution of

* The article "Atmospheric Electricity," by Chree, in the 'Encyclopædia Britannica,' 11th edition, or that by Gerdien in the 'Handbuch der Physik,' vol. 4, p. 687, or Mache and von Schweidler's 'Atmospherische Elektrizität,' chap. I, may be consulted for accounts of the methods used and the results obtained. For recent results obtained at Kew, Chree ('Phil. Trans.,' A, vol. 206, p. 299 (1906), and A, vol. 215, p. 133 (1915)) and Dobson ('Proc. Phys. Soc. Lon.,' vol. 26, p. 334 (1914)) may be consulted.

† Benndorf ('Wiener Ber.,' vol. 109, p. 923 (1900), vol. 115, p. 425 (1906)) has determined with the same simplifications the changes in the vertical potential gradients near a long plateau with rounded edges, a circular plateau, and an ellipsoidal column. Sir J. Larmor and J. S. B. Larmor ('Roy. Soc. Proc.,' A, vol. 90, p. 312 (1914)) give diagrams of potential surfaces, etc., for an ellipsoidal column and an earth-connected sphere.

potential near the middle of their length may be neglected. The walls are further taken to be vertical, and in the case of more than one to be parallel to each other. Roofs are taken to be horizontal.

§ 2. *Case I.*—A long thin vertical wall projects from a horizontal surface above which, at a considerable distance from the wall, the potential gradient is constant. To find the distribution of potential near the wall.

Take the z plane, where $z = x+iy$, vertical and perpendicular to the wall, the x axis along the horizontal plane, and the y axis up the wall. If h is the height of the wall, the Schwarzian transformation $dz/dw = hw(w^2-\alpha^2)^{-\frac{1}{2}}/\alpha$, where $w = u+iv$, converts the x and y axes into the axis of u in the w plane and the first quadrant in the former into the first two quadrants in the latter plane. Integrating, we have

$$z/h = (w^2/\alpha^2-1)^{\frac{1}{2}} \quad \text{or} \quad w^2/\alpha^2 = z^2/h^2+1. \tag{1}$$

If P is a point in the air whose bi-polar co-ordinates are $r\,\theta$, $r'\theta'$, from the top of the wall and from the image of the top in the plane respectively, the last equation gives us at P

$$u = \sqrt{(rr')} \,.\, \cos\tfrac{1}{2}(\theta+\theta'), \qquad v = \sqrt{(rr')} \,.\, \sin\tfrac{1}{2}(\theta+\theta'). \tag{2}$$

Thus the potential v at any point whose bi-polar co-ordinates are given is easily calculated.

To calculate the potential at a point whose co-ordinates are x, y, or to draw the surfaces of equal potential, it is more convenient to use equation (1), which gives on equating separately the real and unreal parts

$$(u^2-v^2)/\alpha^2 = (x^2-y^2+h^2)/h^2 \qquad uv/\alpha^2 = xy/h^2,$$

and on eliminating u,

$$(\alpha^2/h^4)(x^2y^2/v^2) - v^2/\alpha^2 = (x^2-y^2+h^2)/h^2.$$

Thus the potential at the point x, y, is given by

$$v^2 = (\alpha^2/2h^2)\{\sqrt{[(x^2-y^2+h^2)^2+4x^2y^2]}-(x^2-y^2+h^2)\} \tag{3}$$

and the factor by which the observed potential v at the point x, y, should be multiplied to give the potential at the height y above an infinite plane is

$$\frac{y}{(\alpha/\sqrt{2}h)\sqrt{\{\sqrt{[(x^2-y^2+h^2)^2+4x^2y^2]}-(x^2-y^2+h^2)\}}}.$$

The equipotential lines are readily drawn from the equation

$$y^2 = \frac{h^2+x^2+h^2v^2/\alpha^2}{1+\alpha^2x^2/h^2v^2}. \tag{4}$$

They are shown in fig. 1* for the case $h = 1$, $\alpha = 1$.

* I have to thank Lieut. B. Barnes, 10th East Surreys, one of my senior students, for drawing these curves.

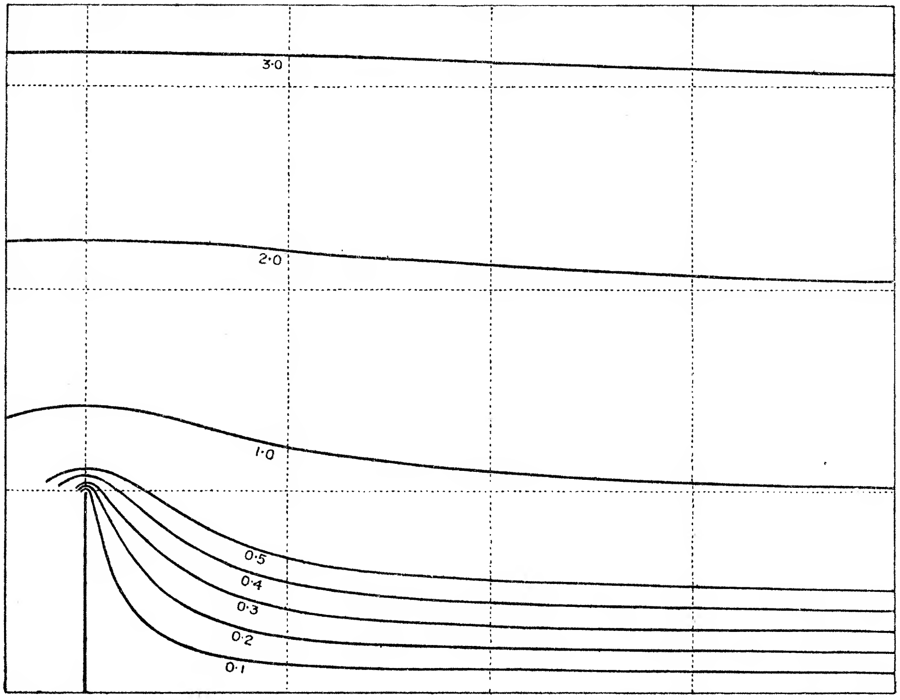

FIG 1.—Section of the equipotential surfaces near the middle of a long thin wall.

§ 3. If x is constant we have, since $w = (\alpha/h)\sqrt{(z^2+h^2)}$,

$$\left(\frac{\partial u}{\partial y}\right)_x + i\left(\frac{\partial v}{\partial y}\right)_x = \frac{\alpha}{h}\frac{-y+ix}{\sqrt{(x^2-y^2+h^2+2ixy)}}.$$

If, further, $y = 0$, then

$$\left(\frac{\partial u}{\partial y}\right)_x = 0 \quad \text{and} \quad \left(\frac{\partial v}{\partial y}\right)_x = \frac{\alpha}{h}\frac{x}{\sqrt{(x^2+h^2)}} = \frac{\alpha}{h} \tag{5}$$

when x is infinitely large.

The last two equations show that the ratio of the vertical potential gradient at a point on the ground to the normal vertical gradient is equal to the cosine of the angle subtended at the point by the height of the wall.

The following Table* shows the vertical gradient at points on the ground whose distances from the foot of the wall are given in terms of the height of the wall.

* This Table and the corresponding ones on pp. 445 and 449 are added at the suggestion of Dr. Chree so as to be available for discussion of the effects of vertical potential gradients on plants and animals.

Table of Ratios of Vertical Gradient to the Normal Vertical Gradient at Points on the Ground near the Middle of a Long Thin Vertical Wall.

$\frac{\text{Distance from foot}}{\text{Height}}$.	$\frac{\text{Vertical gradient}}{\text{Normal}}$.	$\frac{\text{Distance from foot}}{\text{Height}}$.	$\frac{\text{Vertical gradient}}{\text{Normal}}$.
0·0	0·0	1·5	0·832
0·2	0·196	2·0	0·894
0·4	0·371	3·0	0·949
0·6	0·515	4·0	0·970
0·8	0·625	5·0	0·981
1·0	0·707	10·0	0·995

§ 4. If y is constant we have in the same way

$$\left(\frac{\partial u}{\partial x}\right)_y + i\left(\frac{\partial v}{\partial x}\right)_y = \frac{\alpha}{h}\frac{x+iy}{\sqrt{(x^2-y^2+h^2+2ixy)}}.$$

At $x=0$ this gives, if y is less than h,

$$\left(\frac{\partial u}{\partial x}\right)_y = 0, \quad \left(\frac{\partial v}{\partial x}\right)_y = \frac{\alpha}{h}\frac{y}{\sqrt{(h^2-y^2)}}. \tag{6}$$

The horizontal potential gradient at y' on the wall is therefore identical with the normal vertical gradient if

$$y'^2 = h^2 - y'^2,$$

that is if $$y' = h/\sqrt{2} = 0{\cdot}707h.$$

As we proceed outwards from the wall the horizontal potential gradient decreases, but at the height $0{\cdot}707h$ we may move outwards a distance $0{\cdot}1h$ without the potential being 1 per cent. less than that which would have been found at the point if the horizontal gradient had remained constant and equal to the normal vertical gradient. At a distance $0{\cdot}25h$ from the wall the potential is 5 per cent. less than the normal gradient would give.

At larger distances from the wall the change of potential should be calculated directly from either of the expressions (2) and (3) for it in terms of the co-ordinates of the point.

§ 5. *Case II.*—A long vertical retaining wall separates from each other two horizontal plane surfaces over which the potential gradient at a considerable distance from the wall is the same and independent of height above the planes. To determine the distribution of potential near the wall.

Taking the z plane, where $z = x+iy$, vertical and perpendicular to the retaining wall, the x axis along the lower plane, and the y axis up the surface of the wall, we have the Schwarzian transformation

$$dz/dw = a\sqrt{\{(w+\alpha)/(w-\alpha)\}}/\alpha,$$

which converts the lower boundary of the atmosphere in the z plane into the axis of u in the w plane, where $w = u+iv$.

Writing $w/\alpha = \cosh\zeta$, the equation becomes $z = a\int(\cosh\zeta+1)\,d\zeta$, the integral of which is

$$z = a(\sinh\zeta+\zeta). \tag{7}$$

Expanding and separating real and unreal parts, we have

$$\left.\begin{aligned} u/\alpha &= \cosh\xi\cos\eta, & v/\alpha &= \sinh\xi\sin\eta \\ \text{and}\quad x/a &= \sinh\xi\cos\eta+\xi, & y/a &= \cosh\xi\sin\eta+\eta \end{aligned}\right\}. \tag{8}$$

If $v = 0$ we must have either $\xi = 0$ or $\eta = 0$ or π.

In the first case $x = 0$, in the second $y = 0$, in the last $y/a = \pi$. The height h of the wall is therefore equal to $a\pi$, that is $a = h/\pi$, and the equations connecting w and z may be written

$$w/\alpha = \cosh\zeta, \quad \pi z/h = \sinh\zeta+\zeta. \tag{9}$$

To find the potential v at a given point x, y, it is necessary to solve for ξ, η, the second set of transcendental equations (8) or (9), and substitute the values of ξ and η in the equation for v.

To draw the equipotential surfaces we use the equations:—

$$\left.\begin{aligned} \pi(x/h) &= (v/\alpha)\cot\eta+\operatorname{argsinh}(v/\alpha\sin\eta) \\ \pi(y/h) &= \sqrt{[(v^2/\alpha^2)+\sin^2\eta]}+\eta \end{aligned}\right\}, \tag{10}$$

and assign to η all values from 0 to π and to v values exceeding 0.

The curves obtained are shown in fig. 2 for the case in which the height of the equipotential surface of potential unity is equal to that of the wall, that is $\alpha = 1/\pi$, and $h = 1$.

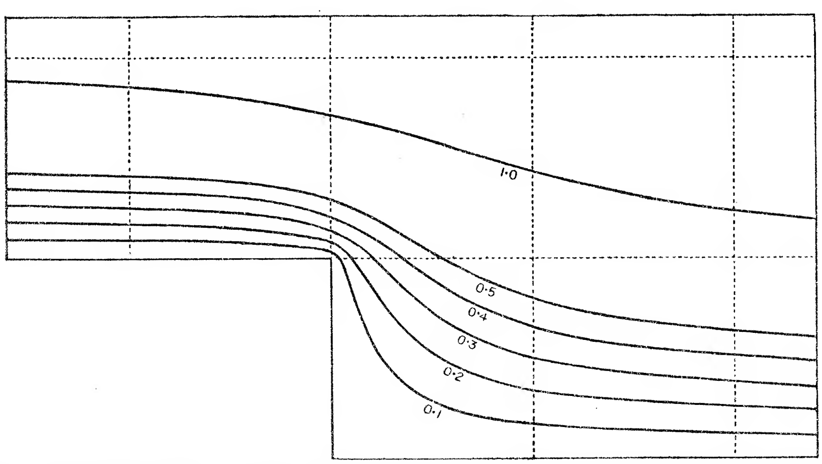

FIG. 2.—Section of the equipotential surfaces near the middle of a long retaining wall.

On differentiating equations (9) we have

$$\left.\begin{aligned}(1/\alpha)\,du &= \sinh\xi\cos\eta\,d\xi - \cosh\xi\sin\eta\,d\eta \\ (1/\alpha)\,dv &= \cosh\xi\sin\eta\,d\xi + \sinh\xi\cos\eta\,d\eta \\ (\pi/h)\,dx &= \cosh\xi\cos\eta\,d\xi - \sinh\xi\sin\eta\,d\eta + d\xi \\ (\pi/h)\,dy &= \sinh\xi\sin\eta\,d\xi + \cosh\xi\cos\eta\,d\eta + d\eta\end{aligned}\right\}. \qquad (11)$$

§ 6. If x is constant we have

$$\cosh\xi\cos\eta\,d\xi - \sinh\xi\sin\eta\,d\eta + d\xi = 0.$$

Thus $$(1/\alpha)\,dv = \left(\frac{\cosh\xi\sinh\xi\sin^2\eta}{\cosh\xi\cos\eta+1} + \sinh\xi\cos\eta\right)d\eta,$$

and $$(\pi/h)\,dy = \left(\frac{\sinh^2\xi\sin^2\eta}{\cosh\xi\cos\eta+1} + \cosh\xi\cos\eta + 1\right)d\eta.$$

Hence the vertical potential gradient at the point corresponding to ξ, η, is given by

$$\begin{aligned}\left(\frac{\partial v}{\partial y}\right)_x &= \frac{\alpha\pi}{h}\frac{\sinh\xi}{\cosh\xi+\cos\eta} \qquad (12) \\ &= \frac{\alpha\pi}{h}\frac{\sinh\xi}{\cosh\xi+1} \qquad \text{if } \eta = 0, \\ &= \frac{\alpha\pi}{h}\frac{\sinh\xi}{\cosh\xi-1} \qquad \text{if } \eta = \pi.\end{aligned}$$

In each case $\left(\frac{\partial v}{\partial y}\right)_x = \frac{\alpha\pi}{h}$ if ξ is large.

The vertical potential gradient is therefore $\alpha\pi/h$ over both planes at considerable distances from the retaining wall. At smaller distances it is less than the normal over the lower and greater than it over the upper plane.

Table of $\frac{\text{Vertical Gradient}}{\text{Normal Vertical Gradient}}$, near the Middle of a Long Retaining Wall or Flat-roofed Building.

Lower surface.		Upper surface.	
$\frac{\text{Distance from foot}}{\text{Height}}$.	$\frac{\text{Gradient}}{\text{Normal}}$.	$\frac{\text{Distance from top}}{\text{Height}}$.	$\frac{\text{Gradient}}{\text{Normal}}$.
0·0	0·0	0·0	∞
0·127	0·100	0·056	2·16
0·258	0·198	0·200	1·58
0·393	0·292	0·518	1·31
0·537	0·380	2·23	1·11
0·690	0·462	22·0	1·01
1·154	0·636		
1·79	0·761		
4·13	0·909		
25·2	0·987		

The preceding Table gives in terms of the normal gradient the vertical gradient at points whose distances from the foot and top of the retaining wall on each of the horizontal surfaces are expressed in terms of the height of the wall.

§ 7. If y is constant we have

$$\sinh\xi \sin\eta \, d\xi + (\cosh\xi \cos\eta + 1)\, d\eta = 0.$$

Thus
$$(1/\alpha)\, dv = \left(\cosh\xi \sin\eta - \frac{\sinh^2\xi \cos\eta \sin\eta}{\cosh\xi \cos\eta + 1}\right) d\xi,$$

and
$$(\pi/h)\, dx = \left(\cosh\xi \cos\eta + 1 + \frac{\sinh^2\xi \sin^2\eta}{\cosh\xi \cos\eta + 1}\right) d\xi.$$

Hence the horizontal potential gradient at the point corresponding to ξ, η, is given by

$$\left(\frac{\partial v}{\partial x}\right)_y = \frac{\alpha\pi}{h} \frac{\sin\eta}{\cosh\xi + \cos\eta}$$

$$= \frac{\alpha\pi}{h} \frac{\sin\eta}{1 + \cos\eta}; \quad \text{if } \xi = 0, \text{ that is when } x = 0. \tag{13}$$

It will be noticed that the ratio of the vertical to the horizontal gradient at the point corresponding to ξ, η, is $\sinh\xi / \sin\eta$.

The horizontal potential gradient close to the wall will be identical with the vertical gradient over the planes at great distances from the wall if

$$\sin\eta = \cos\eta + 1,$$

that is if
$$\eta = \pi/2.$$

The point y' on the wall corresponding to $\xi = 0$, $\eta = \pi/2$, is given by

$$y' = h(\sin\eta + \eta)/\pi = h(1 + \pi/2)/\pi.$$

Hence the point on the wall at which the horizontal gradient outwards is equal to the normal vertical gradient is at a height equal to $(1 + \pi/2)/\pi$ of the total height of the wall, that is to 0·818 of the total height.

As we proceed outwards from the wall at this height the horizontal potential gradient decreases, but at a distance from the wall not exceeding 0·1 of the height of the wall the potential is less than 2 per cent. smaller than it would be if the horizontal gradient had been equal to the normal vertical gradient for the whole distance.

§ 8. *Case III.*—Two long thin vertical walls parallel to each other rise to the same height above a horizontal plane. To find the distribution of potential in the space between the walls.

To simplify the calculation we shall take the walls as being two consecutive walls of a regular series extending on both sides to infinity. Taking the z plane, where $z = x+iy$, vertical and perpendicular to the series of planes, the x axis along the horizontal plane, and the y axis vertical through the point on the x axis half way between the planes, the transformation $\sin w = \alpha \sin(z/a)$,* where α and a are constants, α being less than unity, converts the lower boundary of the atmosphere into the axis of u in the w plane, where $w = u+iv$.

Expanding the circular functions we have

$$\left.\begin{aligned} \sin u \cosh v &= \alpha \sin(x/a) \cosh(y/a) \\ \text{and} \qquad \cos u \sinh v &= \alpha \cos(x/a) \sinh(y/a) \end{aligned}\right\}. \tag{14}$$

If $v = 0$, $\cos(x/a)\sinh(y/a) = 0$, and $\sin u = \alpha\{\sin(x/a)\cosh(y/a)\}$. Thus either $y = 0$ or $(x/a) = \pm(2n+1)\pi/2$, where n is an integer.

If $y = 0$, $\sin u = \alpha \sin(x/a)$, and u and x increase from zero together till $x = a\pi/2$ and $u = \arcsin\alpha$.

If $x/a = \pi/2$, $\sin u = \alpha\cosh(y/a)$, and y increases from zero to $a \operatorname{argcosh} 1/\alpha$, while u increases from $\arcsin\alpha$ to $\pi/2$.

The distance δ of the walls apart is therefore $= a\pi$, and their height h is $a \operatorname{argcosh}(1/\alpha)$. Hence $a = \delta/\pi$, $\alpha = 1/\cosh(\pi h/\delta)$, and the equation connecting w and z may be written

$$\sin w = \sin(\pi z/\delta)/\cosh(\pi h/\delta). \tag{15}$$

The equation to the equipotential lines is therefore

$$\frac{\sin^2(\pi x/\delta)\cosh^2(\pi y/\delta)}{\cosh^2 v} + \frac{\cos^2(\pi x/\delta)\sinh^2(\pi y/\delta)}{\sinh^2 v} = \cosh^2(\pi h/\delta), \tag{16}$$

a quadratic in $\sinh^2 v$ and $\cosh^2 v$ from which v at any point x, y, may be calculated and the reducing factor to convert readings taken at x, y, into readings in the open may be found.

In drawing the equipotential curves it is best to use the equation in the form

$$\cosh\frac{\pi y}{\delta} = \cosh v \sqrt{\left(1 + \frac{\sinh^2(\pi h/\delta)}{1+\cos^2(\pi x/\delta)/\sinh^2 v}\right)}.$$

* The somewhat more general transformation $\sin(w/\beta) = \alpha \sin(z/a)$ may be treated in the same way throughout.

The equipotential curves are shown in fig. 3 for the case $\delta = \pi h$.

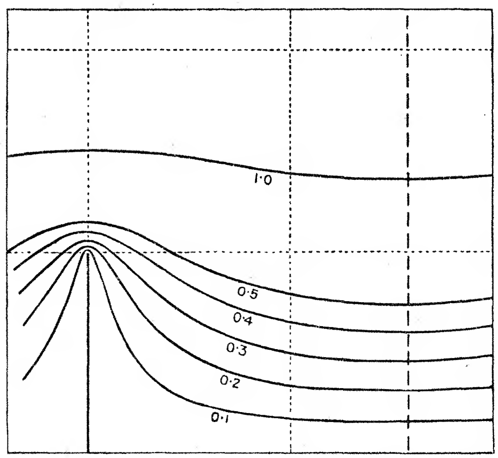

Fig. 3.—Section of the equipotential surfaces near the middle of one of a series of thin parallel walls. The thick dotted vertical line is midway between two planes.

§ 9. If x is constant we have, from equation (15)

$$\left(\frac{\partial u}{\partial y}\right)_x + i\left(\frac{\partial v}{\partial y}\right)_x = i\frac{\pi}{\delta}\frac{\cos\left[\pi(x+iy)/\delta\right]}{\sqrt{\{\cosh^2(\pi h/\delta)-\sin^2\left[\pi(x+iy)/\delta\right]\}}}.$$

At $x = 0$ this reduces to

$$\left(\frac{\partial u}{\partial y}\right)_x = 0, \quad \text{and} \quad \left(\frac{\partial v}{\partial y}\right)_x = \frac{\pi}{\delta}\frac{\cosh(\pi y/\delta)}{\sqrt{[\cosh^2(\pi h/\delta)+\sinh^2(\pi y/\delta)]}}.$$

For large values of y this gives the normal vertical potential gradient $= \pi/\delta$. At $y = 0$ it gives

$$\left(\frac{\partial v}{\partial y}\right)_x = \frac{\pi}{\delta}\frac{\cos(\pi x/\delta)}{\sqrt{[\cosh^2(\pi h/\delta)-\sin^2(\pi x/\delta)]}},$$

that is

$$\left(\frac{\partial v}{\partial y}\right)_x = \frac{\pi}{\delta}\frac{1}{\sqrt{[\sinh^2(\pi h/\delta)/\cos^2(\pi x/\delta)+1]}}. \qquad (17)$$

At the point on the ground half-way between the planes

$$\left(\frac{\partial v}{\partial y}\right)_x = \frac{\pi}{\delta}\frac{1}{\cosh(\pi h/\delta)}. \qquad (18)$$

Thus the ratio of the vertical gradient on the ground half-way between the planes to the normal vertical gradient is $1/\cosh(\pi h/\delta)$.

The following Table gives the values of this ratio for different values of the ratio of height of planes to distance apart.

Table of $\dfrac{\text{Vertical Gradient}}{\text{Normal Vertical Gradient}}$ on the Ground Midway between Two Vertical Planes for Different Ratios of Height of Planes to Distance apart.

$\dfrac{\text{Height}}{\text{Distance apart}}$.	Vertical gradient.	$\dfrac{\text{Height}}{\text{Distance apart}}$.	Vertical gradient.	$\dfrac{\text{Height}}{\text{Distance apart}}$.	Vertical gradient.
0 ·0	1 ·0	0 ·382	0 ·552	1 ·273	0 ·0366
0 ·064	0 ·980	0 ·572	0 ·322	1 ·592	0 ·0135
0 ·127	0 ·925	0 ·763	0 ·180	1 ·910	0 ·0050
0 ·191	0 ·844	0 ·954	0 ·099		

The vertical gradient at other points on the ground between the planes is given by the equation (17) above, and is tabulated below for several values of the ratio of height of the planes to distance apart.

Table of $\dfrac{\text{Vertical Potential Gradient}}{\text{Normal Potential Gradient}}$ on the Ground between Two Vertical Planes.

$\dfrac{\text{Distance of point}}{\text{Distance of planes apart}}$.		$\dfrac{\text{Height of planes}}{\text{Distance apart}}$.			
From plane.	From middle.	0 ·064.	0 ·191.	0 ·382.	0 ·763.
0 ·0	0 ·5	0 ·0	0 ·0	0 ·0	0 ·0
0 ·1	0 ·4	0 ·844	0 ·445	0 ·201	0 ·056
0 ·2	0 ·3	0 ·946	0 ·679	0 ·365	0 ·107
0 ·3	0 ·2	0 ·970	0 ·783	0 ·473	0 ·146
0 ·4	0 ·1	0 ·978	0 ·831	0 ·533	0 ·172
0 ·5	0 ·0	0 ·980	0 ·844	0 ·552	0 ·180

These figures are sufficient to show how great is the effect of the walls on the vertical gradient on the ground between them. They may be taken as representing with a fair degree of accuracy the vertical gradient on the ground in a street with buildings on each side of it.

§ 10. If y is constant we have

$$\left(\frac{\partial u}{\partial x}\right)_y + i\left(\frac{\partial v}{\partial x}\right)_y = \frac{\pi}{\delta}\frac{\cos\left[\pi(x+iy)/\delta\right]}{\sqrt{\{\cosh^2(\pi h/\delta)-\sin^2\left[\pi(x+iy)/\delta\right]\}}}.$$

At $x = \delta/2$ this becomes

$$\left(\frac{\partial u}{\partial x}\right)_y + i\left(\frac{\partial v}{\partial x}\right)_y = \frac{\pi}{\delta}\frac{-i\sinh(\pi y/\delta)}{\sqrt{\{\cosh^2(\pi h/\delta)-\cosh^2(\pi y/\delta)\}}}.$$

If y is less than h, this gives

$$\left(\frac{\partial u}{\partial x}\right)_y = 0, \quad \text{and} \quad \left(\frac{\partial v}{\partial x}\right)_y = -\frac{\pi}{\delta}\frac{\sinh(\pi y/\delta)}{\sqrt{\{\cosh^2(\pi h/\delta)-\cosh^2(\pi y/\delta)\}}}. \qquad (19)$$

If the horizontal potential gradient at y' on the wall is to be equal to the normal vertical gradient π/δ, we must have

$$\sinh^2(\pi y'/\delta) = \cosh^2(\pi h/\delta)-\cosh^2(\pi y'/\delta),$$

that is
$$\sinh(\pi y'/\delta) = (1/\sqrt{2})\sinh(\pi h/\delta).$$

When the height h becomes small compared to the distance apart δ of the planes, this gives the distance y' up the plane as 0·71 times the height, as was found by direct calculation.

The following Table gives the relation between the ratio of the height of the planes to their distance apart and the fraction of the height at which the horizontal potential gradient outwards is equal to the normal vertical potential gradient over a plane surface.

$\frac{\text{Height}}{\text{Distance apart}}$.	Fraction of height.	$\frac{\text{Height}}{\text{Distance apart}}$.	Fraction of height.
0·0	0·707	0·445	0·790
0·064	0·708	0·509	0·807
0·127	0·717	0·572	0·822
0·191	0·727	0·636	0·836
0·254	0·741	—	—
0·318	0·757	0·795	0·864
0·382	0·773	0·954	0·886

As the planes approach each other the potential surfaces near their summits become more nearly horizontal, so that the potential gradient is more nearly vertical than horizontal.

§ 11. It follows from the previous work that where plane horizontal surfaces of considerable extent are not available for the determination of the normal vertical potential gradient in the atmosphere, observations in the neighbourhood of buildings can be utilised, the value of the reducing factor being calculated from the forms of the buildings in the simple cases dealt with. In many cases observations of the horizontal potential gradient outwards from the walls of buildings which are not too close together may be made, and, if the position of the point of observation is properly chosen, the horizontal gradient observed will be identical with the normal vertical gradient over a horizontal surface. For a long wall of a building with a flat roof or with a parapet, the horizontal gradient outwards should be measured

at a point near the middle of the length of the wall and at a distance up it which is generally about three-quarters of the height. The horizontal gradient for a distance outwards not exceeding 1/10 the height of the wall will not differ by more than 2 per cent. from the normal vertical gradient over a large horizontal area.

On the Enhanced Series of Lines in Spectra of the Alkaline Earths.

By W. M. Hicks, Sc.D., F.R.S.

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The problem of the limits and numerical relations between the lines of the enhanced series of doublets in the alkaline earths has for long been a difficulty to spectroscopists. Ritz* in 1908 gave arrangements for the Sharp series from Mg to Ra inclusive, and proposed series formulæ for Ca, Sr, Ba, in which alone he had three lines from which to calculate the constants. The absence of extra lines rendered it impossible to test his formulæ, but the values of the constants obtained for his formulæ were quite out of line with those of the analogous constants in other series, and produced an instinctive doubt as to whether it gave the correct relation. It is now possible to test his limits by considering whether the denominator differences which give the observed separations have any relation to the oun or not. The result of this consideration is definitely adverse. In none of the three is it possible to make the differences multiples of the oun without supposing observation errors in the doublet separations which are quite inadmissible; and even then in the cases of Ca and Ba by taking odd multiples of δ_1, which is never the case for S doublets in any other known series.

There can be little doubt but that Fowler† has at last settled this question by taking the Rydberg numerator constant to be 4N in place of N, thus combining in one set lines which on the old supposition would be arranged in two series, depending on Sharp and Principal sequences. The object of the present note is the determination of the connection of these series with certain laws which have been arrived at in previous communications‡ to this Society

* 'Phys. Zeitschr.,' vol. 16, p. 521.

† 'Phil. Trans.,' A, vol. 214, p. 225 (1914).

‡ 'Phil. Trans.,' A, vol. 210, p. 57 (1909); A, vol. 212, p. 33 (1912); A, vol. 213, p. 323 (1913)—referred to in the following as (I), (II), and (III) respectively.